Broadening the Descriptors of van Hieles' Levels 2 and 3

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Despite controversy, the van Hiele levels continue to be used as an important framework for developing geometry teaching programs and for interpreting students' understanding of geometrical ideas. However, as they stand, the level descriptors offer a restrictive base. These cause problems when questions are posed outside of the direct notions of properties of figures, class inclusions and deduction about which the Theory is explicit This paper is an initial attempt to broaden the level descriptors in a way that is consistent with the original ideas of the theory but which allows for more inclusive criteria. To assist in this process ideas drawn from the SOLO Taxonomy are employed.

Introduction

The van Hiele Theory has been the focus of continued research attention since the late 70s. In particular, many theses, reports and articles have been published concerning issues related to verification and exploration of the five hierarchical levels, and their associated characteristics, that are an important basis of the Theory.

The levels have proved a useful tool in (i) identifying problems in students' understanding of certain geometrical concepts, (ii) evaluating the structure or development of geometric content in secondary text books, and (iii) guiding the development of syllabuses. However, the theory has not been without controversy. Some (Gutiérrez *et al.* 1991) have challenged the discontinuous nature of the levels. Others have queried the apparently simplistic one-dimensional nature of the levels and believe the theory fails to describe the diversity evident in students' behaviour. Despite these forms of criticism, there is considerable empirical support for the levels (Clements and Battista, 1992).

The van Hiele Theory hypothesises five levels of thinking identified as Levels 1 to 5. Of interest to this paper are the first four levels. These are described below with the modification to Level 2 suggested by Pegg (1995).

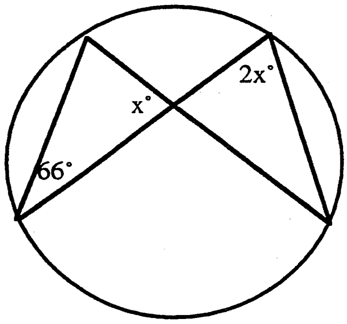
Level 1:	Figures are identified according to their overall appearance.
Level 2A:	Figures are identified in terms of a single property (usually sides).
Level 2B:	Figures are identified in terms of properties which are seen as independent of one another.
Level 3:	Relationships between previously identified properties of a figure are established as well as relationships between the figures themselves.
Level 4:	Deduction is understood and students can develop logical proofs. Multiple definitions in terms of minimum properties can be provided.

There are at least two important reasons for splitting van Hiele's Level 2. Many students, especially up to the first or second year of secondary education, respond at this level when asked to describe a figure. Second, attaining Level 2A represents a culmination in the thinking process of the development of a single concept or property. As such, it represents an important interface between the visual/intuitive thinking at Level 1 and the identification of several isolated concepts/properties at Level 2B.

Background

One issue that has confronted the writer is that the current level descriptors are narrow and not easily generalisable to a range of question types common in school geometry. When one considers some of the typical questions posed of students in the junior secondary school there does not appear to be much guidance from the van Hiele Theory in allocating levels. Reference to van Hiele writings or the two most commonly used tests, i.e., those devised by Usiskin (1982) and Mayberry (1981) do not help. For example, the following six questions, taken from the 1985 and 1986 State School Certificate Reference Test of New South Wales, typify the concern.

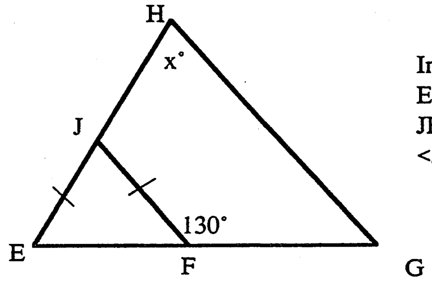
Question 1



x = ?

A. 33 B. 38 C. 48 D. 57

Question 2

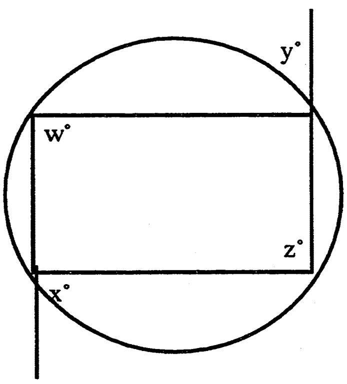


In the diagram
EJ = JF
JF || HG
<JFG = 130°

x = ?

A. 50 B. 80 C. 100 D. 130

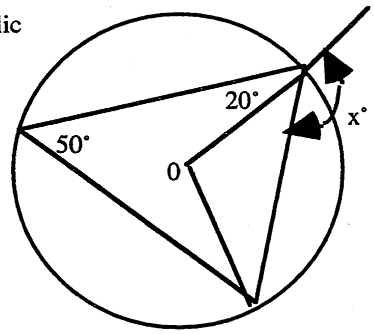
Question 3



The diagram shows a cyclic quadrilateral.
Which statement below is always true?

A. z = w
B. x = w
C. x + y = 180
D. x + w = 180

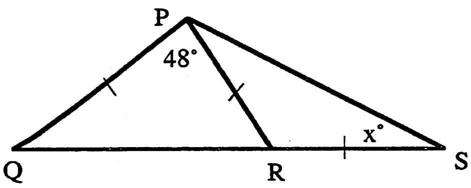
Question 4



The diagram shows a circle centre 0.
x = ?

A. 110
B. 120
C. 130
D. 140

Question 5



x =

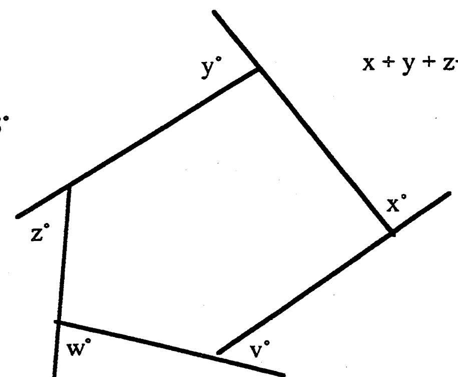
A. 33 B. 42 C. 57 D. 66

Question 6

PQ = PR = RS

Angle QPR = 48°

Angle RSP = x°



x + y + z + w + v = ?

A. 180
B. 360
C. 540
D. 720

These items were given, as part of general mathematics examinations to some 35,000 students (who represent 40% of the Year 10 (16 year old) student population) doing the advanced mathematics course in the state of New South Wales, Australia. Significantly, a consistent pattern emerged in the nature of responses. This showed (i) students who were correct on Questions 2 and 5 (approx 75% of the candidature) were also able to handle correctly and consistently typical Level 2 items, and (ii) students who were correct on Questions 1, 3 and 6 (approx 45% of the candidature) performed correctly and consistently on more typical Level 3 items. Approximately 60% of the candidates were correct with Question 4.

While there was clearly a close relationship between these items and the two van Hiele levels, van Hiele's descriptors give little guidance to support coding the nature of the thinking associated with these five questions. Hence, the focus of this paper is on considering how the descriptors of the van Hiele levels, as previously provided, can be expanded so that while they remain true to the original descriptions they can also be inclusive of, and able to categorise, the type of questions posed above.

The SOLO Taxonomy

One approach to addressing the issue of more inclusive level descriptions is to refer to a theory that is seen to have some sympathy with the van Hiele Theory. The SOLO Taxonomy (Biggs & Collis, 1982, 1991) has been identified by several writers (e.g., Jurdak, 1989; Olive, 1991; Pegg & Davey, 1989) as having strong similarities with the van Hiele Theory, despite some apparent philosophical differences.

The SOLO Taxonomy, as with the van Hiele Theory, has its roots in the Piagetian tradition and both theories are relevant to and designed to facilitate school-learning activities, albeit in different ways. The van Hiele levels are a series of signposts of cognitive growth reached through a teaching/learning process as opposed to some biological maturation. SOLO, however, is particularly applicable to judging the quality of instructional dependent tasks. It is concerned with evaluating the quality of students' responses to various stimulus items. While it is possible to set questions which encourage a response at a particular level, it is students' attempts at an item that are of paramount interest as well as the many natural groupings of answer types. This focus represents an important departure from, say, Bloom's Taxonomy where levels have an *a priori* quality and students are deemed either successful or not. With SOLO, the data (students' responses) are treated polychotomously (as opposed to dichotomously, i.e., true or false) and the categorisation of answers into multiple groupings with similar characteristics reflects various stages of cognitive growth.

This represents a philosophic shift from van Hiele's (and Piaget's) ideas, as the levels describe responses, not people. With SOLO, a response provides a measure of a student's attainment at a particular time and in a particular circumstance, it does not, necessarily, determine some stable stage of cognitive functioning. This approach overcomes the décalage problem identified by Piaget in which a person may respond at a different level to the same (or similar) tasks from one testing episode to the next.

Fortunately, this difference in focus does not represent a fatal flaw to any comparison of level descriptions across the two frameworks. The main difference between them is manifested in the conclusions that are drawn about the overall nature of a students' level of thinking. This contrasting view is not the focus of this paper and useful comparisons about levels can be made without confronting this issue.

A SOLO classification combines two aspects. The first of these is the mode of functioning and, the second, a level of quality of response within the targeted mode. Of relevance here is the mode referred to as concrete symbolic and the three levels within the mode referred to as unistructural, multistructural and relational.

In the concrete symbolic mode a student is capable of using, or learning to use, a symbol system, such as a written language and number notation. The important feature of this mode is that there is an empirical referent available. This is the most common mode addressed in learning in the upper primary and secondary school. This mode becomes available after an individual has progressed to a certain level of attainment within the ikonic mode. (The ikonic mode is where a person internalises outcomes in the form of images. It is in this mode that a child develops words and images that can stand for objects and events.) Hence, early responses in the concrete symbolic mode carry with them the need to base judgments on observable, physical experiences that make sense to the real-world understanding of the person.

The three levels within the concrete symbolic (C.S.) mode (Biggs & Collis, 1991), represent a growth from the more 'concrete' to the more 'abstract'. Brief descriptions of these levels are:

The *unistructural level* of response is one that contains, or draws upon, one relevant concept or datum from among all that was available.
The *multistructural level* of response is one that contains several relevant but isolated concepts or data from among all that was available.
The *relational level* of response is one that contains an over-riding linking concept. Alternatively, each relevant concept is woven together to form a coherent structure.

The development encompassed by the three SOLO levels moves from a focus on a simple (one aspect) and more tangible aspect, which is closely aligned with an individual's real-world experience, to the less tangible aspects, namely, relationships between concepts. This growth is, in part, determined by the availability of working memory for the completion of the task. At the unistructural level the student has only to understand the question, relate the question and the answer, and use one concept. The multistructural level response requires a similar ability except the student needs to be able to access a number of concepts. The relational response requires, in addition, an overview of relevant concepts while being able to monitor the process or task from beginning to end, thus allowing for a logically complete conclusion.

Finally, notions of consistency and closure are also strongly related to each level. The former refers to the need felt by individuals to make a response that is not contradictory, either from the perspective of the answer or the data provided. The latter refers to the desire to provide an answer and hence finish the task. These two needs represent opposing forces that impact on the nature or quality of the response.

The link between the level descriptors of SOLO and the van Hiele Theory can be summarised as: unistructural responses (C.S. mode) are consistent with Level 2A thinking; multistructural responses (C.S. mode) with Level 2B thinking; and, relational responses (C.S. mode) with Level 3 thinking.

Results

Level 2A

In terms of the five questions posed at the beginning of the paper, none can be seen as requiring Level 2A thinking. None require an application of a single concept related to a (visually) verifiable situation. Examples of questions in which a unistructural response or Level 2A thinking is required are given below in Figure 1. Here, for those students who have access to the relevant notion, the diagram acts as a cue to prompt a single appropriate concept involving 'real' angle measurements. Further, the result involves a unique number, which can be obtained quickly, and hence the demands on working memory are low.

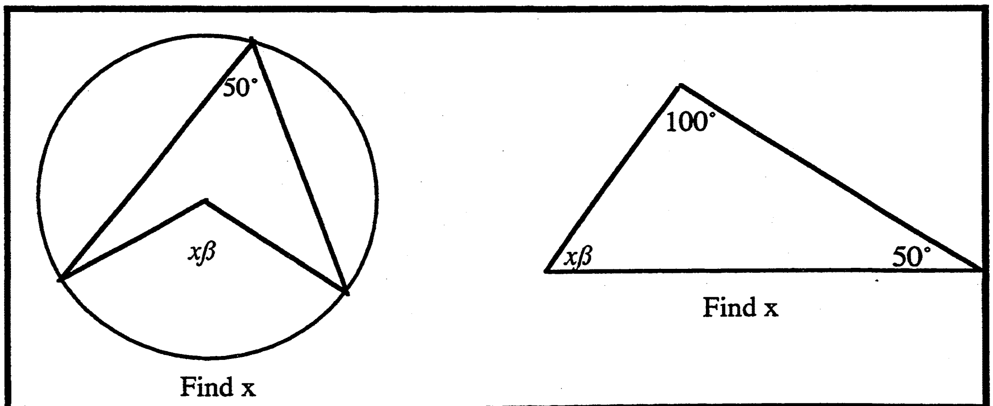


Figure 1: Questions requiring Level 2A thinking

The purpose of these questions is to ascertain whether the respondent has knowledge of the appropriate concept and can understand the question. In both cases a single concept is needed, and real angle measurements are used. A student can 'close' quickly if the demands on the working memory are small. Consistency can be assisted by the visual support given to the answer that would confirm it being unique and 'real' to the student.

Level 2B

The three questions, Questions 2, 4 and 5, each belong to this category. To complete successfully these tasks, the student must employ more than one concept. There is a clear, unique end-point, the use of actual angle measurements ensures a series of 'real', meaningful closures for the students. The questions can be solved without the need of an overview of the questions and the available data. Students are able to move logically along a path from the given angle to the answer without needing to know in advance how they might proceed. Indeed the path to the answer would be unexpected until the completion of the activity.

The reason for the lower correct response rate for Question 4 (60% as opposed to 75%) is related to the distraction caused by the 20° angle. This sets up some interference to the solution process with students attempting to start with this angle or, alternatively, use it part way. Absence of an overview means such information can set a student on an unsuccessful solution path from which they can not recover.

Level 3

The three questions, Questions 1, 3 and 6, each belong to this level. The major advance in thinking appears to be the ability to refrain from seeking individual closures (answers) before proceeding to the next step. This represents an ability to form a generalisation based on previous 'concrete' experiences with specific cases although final uniqueness of the result still needs to be guaranteed. This level represents an ability to work with pronumerals. In each case this usage can be supported by replacement of some unique numbers, which make sense to the student within the questions' context, if the student feels pressured. In both cases, students who are consistently successful at this type of problem have an overview of the elements of the question. They are aware of the solution strategy which relates the use of various concepts and, unlike those who perform at Level 2B, the arrival at the final answer would not come as a surprise.

Conclusion

The above analysis has identified a way forward to expand upon the descriptions offered by van Hiele for his Levels 2 and 3. In essence, this development means, in the case of Level 2, that the characteristic of thinking in terms of independent properties can be interpreted within SOLO as an aspect of a broader thinking category in which concepts are addressed in isolation. These concepts need to have an obvious visual basis and individual closures (answers) must have a strong real-world referent for students. Aspects of processing which occur towards the end of a task appear independent of any initial processing. The individual steps (or closures) leading to a final solution can be performed in sequence without concern given to some general overview. A distractor in the process can cause concerns.

At Level 3, the characteristic of thinking concerns the acceptance and use of relationships between properties and figures. The broadening of this level using SOLO is associated with the ability to have an overview of relevant elements and to form, on this basis, appropriate generalisations. Consequently, for a given task, relevant data are identified and the student can monitor these data. This allows for the reasoning at a later stage in a question to be adjusted in the light of earlier thinking. Also at this level, students have a notion of a generalised number by using algebraic symbols which can stand for 'real' numbers. Hence, they do not, necessarily, replace pronumerals with numbers but students feel secure as they have the option to do this in cases in which they perceive to be more difficult. This ability to work with pronumerals allows students to refrain from the need to calculate particular answers for each step of a problem (a characteristic of Level 2 thinking), and opens the way for relationships between different concepts to be utilised.

Interestingly, support for these latter ideas can be found in van Hiele's own writings. He stated "The differences between the objects of the second and third levels can also be demonstrated by different ways of writing. At the second level, calculation deals with relations between concrete numbers: $4 \times 3 = 12$, $6 + 8 = 14$. At the third level of thinking it deals with generalisation of results: $a \times (b + c) = (a \times b) + (a \times c)$. In these generalisations you do not return to the original objects of the second level, namely the concrete numbers"(1986, p.54).

Here, we can see van Hiele broadening his own level descriptions. Surprisingly, van Hiele chose not to pursue this line of thought. However, with the support of SOLO and the discussion in this paper, this quote can assume a much deeper meaning. For example, at the second level there is a focus on actual numbers, single concepts are involved, and working memory demand is relatively light. At the third level, the working memory demands are heavier, the example given is more abstract in nature and actual numbers are not used. The rule cannot be known by its separate parts only by an overview of all the elements and the structure of the relationships can the pattern be understood. The letters in the equation each represent an entity in their own right, there is no direct concrete referent for the rule although substitution of numbers remains an option which would guarantee uniqueness of outcome.

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